

# Strong Decays of Strange Charmed P-Wave Mesons

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## Abstract

Goldstone boson decays of P-wave  $D_s^{**}$  mesons are studied within the framework of Heavy Hadron Chiral Perturbation Theory. We first analyze the simplest single kaon decays of these strange charmed mesons. We derive a model independent prediction for the width of  $D_{s2}$  and use experimental information on  $D_{s1}$  to constrain the S-wave contribution to  $D_1^0$  decay. Single and double pion decay modes are then discussed and shown to be significantly restricted by isospin conservation. We conclude that the pion channels may offer the best hope for detecting one strange member of an otherwise invisible P-wave flavor multiplet.

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## 1. Introduction

During the past few years, a synthesis of Chiral Perturbation Theory and the Heavy Quark Effective Theory (HQET) has been developed [1–5]. This hybrid effective theory describes the low energy strong interactions between light Goldstone bosons and hadrons containing a heavy quark. Heavy Hadron Chiral Perturbation Theory (HHcPT) has been most widely applied to processes involving charm and bottom hadrons that correspond to ground state mesons and baryons in the nonrelativistic quark model. It is however straightforward to incorporate orbital or radial excitations into the formalism and to study their transitions as well. Experimental information on such excited mesons and baryons is at present much less plentiful than for the lowest lying heavy hadrons. However, data on excited charm hadrons is currently being collected at CLEO [6–8] and Fermilab [9], and future experiments are expected to fill in many of their basic properties. The phenomenology of these new particles will provide valuable tests of several basic HQET ideas.

Of all the possible excited heavy hadrons, P-wave mesons are among the simplest. They are characterized in the quark model as heavy quark-light antiquark bound states carrying one unit of orbital angular momentum. Such mesons have been included into the heavy hadron chiral Lagrangian in refs. [10] and [11]. The Lagrangian was then used to study single and double pion decays of  $D^{0**}$  and  $D^{+**}$  for which the greatest amount of experimental data is currently available. In this article, we would like to extend these previous investigations and focus instead upon  $D_s^{**}$  states. As we shall see, isospin considerations significantly restrict the decays of excited strange charmed mesons and can lead to qualitatively different results.

Our paper is organized as follows. In section 2, we review the incorporation of P-wave mesons into HHcPT. We then focus in section 3 upon  $D_s^{**}$  states and discuss their single kaon decay modes. We derive a model independent prediction for the  $D_{s2}$  width, and we use experimental limits on  $D_{s1}$  to constrain the S-wave component in  $D_1^0$  decays. We then investigate single and double pion decay channels in section 4 and discuss prospects for finding a particular  $D_s^{**}$  state that belongs to a class of P-wave mesons which has never been seen. Finally, we close in section 5 with a summary of our results.

## 2. The Heavy Meson Chiral Lagrangian

We begin by recalling the fields which enter into the heavy meson chiral Lagrangian. The Goldstone bosons resulting from the chiral symmetry breakdown  $SU(3)_L \times SU(3)_R \rightarrow SU(3)_{L+R}$  appear in the pseudoscalar meson octet

$$\boldsymbol{\pi} = \sum_{a=1}^8 \pi^a T^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} \quad (2.1)$$

and are conventionally arranged into the exponentiated matrix functions  $\Sigma = e^{2i\boldsymbol{\pi}/f}$  and  $\xi = e^{i\boldsymbol{\pi}/f}$ . These matrix functions transform under the chiral symmetry group as

$$\begin{aligned} \Sigma &\rightarrow L\Sigma R^\dagger \\ \xi &\rightarrow L\xi U^\dagger(x) = U(x)\xi R^\dagger \end{aligned} \quad (2.2)$$

where  $L$  and  $R$  represent global elements of  $SU(3)_L$  and  $SU(3)_R$  while  $U(x)$  acts like a local  $SU(3)_{L+R}$  transformation. Chiral invariant terms describing Goldstone boson self interactions may be constructed from the fields in (2.2) and their derivatives.

Mesons with quark content  $Q\bar{q}$  absorb and emit light Goldstone bosons with no appreciable change in their four-velocities in the infinite mass limit of their heavy quark constituents  $Q$ . They are consequently represented by velocity dependent fields. We absorb square roots of meson masses into these fields' definitions so that one particle states are normalized as

$$\langle p, s | p', s' \rangle = 2v^0 (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{p}') \delta_{ss'}. \quad (2.3)$$

The velocity dependent fields then have mass dimension 3/2.

The ground state  $J^P = 0^-$  and  $J^P = 1^-$  mesons which result from coupling together the heavy quark and light antiquark spins in an S-wave bound state are annihilated by the pseudoscalar and vector meson operators  $P_i(v)$  and  $P_{i\mu}^*(v)$ . Their individual components are given by  $(P_1, P_2, P_3) = (D^0, D^+, D_s)$  and  $(P_{1\mu}^*, P_{2\mu}^*, P_{3\mu}^*) = (D^{0*}, D^{+*}, D_s^*)$  when the heavy quark inside the meson is taken to be charm. In the infinite quark mass limit, it is useful to combine the degenerate  $J^P = 0^-$  and  $J^P = 1^-$  states into the  $4 \times 4$  matrix field [1,12]

$$H_i(v) = \frac{1 + \not{v}}{2} [-P_i(v)\gamma^5 + P_{i\mu}^*(v)\gamma^\mu]. \quad (2.4)$$

The superfield  $H$  carries a heavy quark spinor index and a separate light antiquark spinor index, and it transforms as an antitriplet under flavor  $SU(3)_{L+R}$  and doublet under spin symmetry  $SU(2)_v$ .

In P-wave excited mesons, the antiquark spin can pair with one unit of orbital angular momentum to form states with light angular momentum  $j_\ell = 1/2$  or  $j_\ell = 3/2$ . Coupling with the heavy quark spin then yields two pairs of two degenerate states. In the first case, the resulting  $J^P = 0^+$  and  $J^P = 1^+$  mesons are annihilated by the operators  $P_i^*(v)$  and  $P'_{i\mu}(v)$ . In the second case, the  $J^P = 1^+$  and  $J^P = 2^+$  mesons are associated with  $P_{i\mu}(v)$  and  $P_{i\mu\nu}^*(v)$ . When the heavy quark is charm, we identify the individual  $SU(3)$  components of all these operators with the excited meson states as follows: <sup>1</sup>

$$\begin{aligned} (P_1^*, P_2^* P_3^*) &= (D_0^0, D_0^+, D_{s0}) \\ (P'_{1\mu}, P'_{2\mu}, P'_{3\mu}) &= (D_1^{0'}, D_1^{+'}, D'_{s1}) \\ (P_{1\mu}, P_{2\mu}, P_{3\mu}) &= (D_1^0, D_1^+, D_{s1}) \\ (P_{1\mu\nu}^*, P_{2\mu\nu}^*, P_{3\mu\nu}^*) &= (D_2^0, D_2^+, D_{s2}). \end{aligned} \tag{2.5}$$

The scalar  $P_i^*$  and axialvector  $P'_{i\mu}$  operators may be assembled into the superfield

$$S_i(v) = \frac{1 + \not{v}}{2} [-P_i^*(v) + P'_{i\mu}(v) \gamma^\mu \gamma^5] \tag{2.6}$$

which is just the parity reversed analog of  $H$ . The axialvector  $P_{i\mu}$  and traceless, symmetric tensor  $P_{i\mu\nu}^*$  operators may be combined into a third superfield [10]

$$T_i^\mu = \frac{1 + \not{v}}{2} [P_i^{*\mu\nu}(v) \gamma_\nu - \sqrt{\frac{3}{2}} P_{i\nu}(v) \gamma^5 (g^{\mu\nu} - \frac{1}{3} \gamma^\nu [\gamma^\mu - v^\mu])]. \tag{2.7}$$

$S$  and  $T^\mu$  transform exactly like  $H$  under  $SU(3)_{L+R}$  and  $SU(2)_v$ .

It is important to note that  $H$ ,  $S$  and  $T^\mu$  obey the following constraints which restrict the form of interactions that one can write down for these fields:

$$\frac{1 + \not{v}}{2} H(v) = H(v) \quad H(v) \frac{1 - \not{v}}{2} = H(v) \tag{2.8a}$$

$$\frac{1 + \not{v}}{2} S(v) = S(v) \quad S(v) \frac{1 + \not{v}}{2} = S(v) \tag{2.8b}$$

$$\frac{1 + \not{v}}{2} T^\mu(v) = T^\mu(v) \quad T^\mu(v) \frac{1 - \not{v}}{2} = T^\mu(v). \tag{2.8c}$$

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<sup>1</sup> In the absence of a universally accepted nomenclature for P-wave mesons, we adopt the convention of labeling states with total angular momentum subscripts and electric charge superscripts. In the course of the text, we also frequently follow the common albeit informal practice of denoting P-wave mesons with double asterisk superscripts.

Multiplication on the left by the projection operator  $P_+ = (1 + \not{v})/2$  simply picks out the two heavy quark degrees of freedom in all the meson superfields. Multiplication on the right by  $P_\pm$  effectively projects out two light degrees of freedom. These two conditions account for a total of four degrees of freedom within  $H$  and  $S$  corresponding to one  $J = 0$  and three  $J = 1$  meson states. The  $T^\mu$  superfield obeys two additional auxiliary constraints [13]

$$v_\mu T^\mu(v) = T^\mu(v) \gamma_\mu = 0 \quad (2.9)$$

that reduce its degrees of freedom to eight.  $T^\mu$  thus precisely accommodates three  $J = 1$  and five  $J = 2$  states.

Interactions involving the meson superfields are further constrained by reparameterization invariance [5,14]. Recall that the decomposition  $p = Mv + k$  of a heavy particle's four-momentum in terms of its four-velocity  $v$  and residual momentum  $k$  is somewhat arbitrary. To  $O(1/M)$ , no physical result should be altered if these parameters are redefined as

$$\begin{aligned} v &\rightarrow v + \epsilon/M \\ k &\rightarrow k - \epsilon \end{aligned} \quad (2.10)$$

where  $v \cdot \epsilon = 0$ . This change of variables leaves the total four-momentum  $p$  invariant and induces only an  $O(1/M^2)$  correction to  $v^2 = 1$ . In Heavy Hadron Chiral Perturbation Theory, the parameter redefinitions generate shifts in the meson superfields

$$\begin{aligned} H &\rightarrow H + \delta H \\ S &\rightarrow S + \delta S \\ T^\mu &\rightarrow T^\mu + \delta T^\mu \end{aligned} \quad (2.11)$$

which are fixed by the constraints in (2.8) and (2.9) and by a superfield normalization condition. For instance, varying the relations  $\not{v}H = H$  and  $H\not{v} = -H$  which follow from (2.8a) yields

$$\begin{aligned} (\not{v} + \frac{\not{\epsilon}}{M})(H + \delta H) &= H + \delta H \\ (H + \delta H)(\not{v} + \frac{\not{\epsilon}}{M}) &= -(H + \delta H). \end{aligned} \quad (2.12)$$

Solving these equations along with the normalization constraint  $\text{Tr}(\overline{H}H) = \text{Tr}[(\overline{H} + \delta\overline{H})(H + \delta H)]$  for  $\delta H$ , one readily deduces that it is proportional to a commutator:

$$\delta H = \left[ \frac{\not{\epsilon}}{2M}, H \right]. \quad (2.13)$$

In a similar manner, we find the variations in the  $S$  and  $T^\mu$  superfields

$$\delta S = \left\{ \frac{\not{\epsilon}}{2M}, S \right\} \quad \text{and} \quad \delta T^\mu = \left[ \frac{\not{\epsilon}}{2M}, T^\mu \right] - \frac{\epsilon_\nu T^\nu}{M} v^\mu. \quad (2.14)$$

The requirement that the effective theory remain invariant under the transformations in (2.10) and (2.11) then forbids certain terms such as  $\text{Tr}(\overline{H} i D_\mu S \gamma^\mu)$  and  $\text{Tr}(\overline{H} i D_\mu T^\mu)$  from appearing in the chiral Lagrangian [10].

With the meson superfields in hand, one can readily write down the leading order effective chiral Lagrangian that describes the low energy interactions between Goldstone bosons and mesons in the infinite heavy quark mass limit. The lowest order terms must be hermitian, Lorentz invariant and parity even. They must also respect the light chiral and heavy quark spin symmetries and be consistent with reparameterization invariance:

$$\begin{aligned} \mathcal{L}_v^{(0)} = \sum_{Q=c,b} \bigg\{ & -\text{Tr}[\overline{H} i v \cdot D H] + \text{Tr}[\overline{S}(i v \cdot D - \Delta M_S) S] + \text{Tr}[\overline{T}_\mu(i v \cdot D - \Delta M_T) T^\mu] \\ & + g_1 \text{Tr}[\overline{H} H \not{\mathbf{A}} \gamma^5] + g_2 \text{Tr}[\overline{S} S \not{\mathbf{A}} \gamma^5] + g_3 \text{Tr}[\overline{T}_\mu T^\mu \not{\mathbf{A}} \gamma^5] \\ & + f_1 \text{Tr}[(\overline{H} S + \overline{S} H) v \cdot \mathbf{A} \gamma^5] + f_2 \text{Tr}[(\overline{S} T^\mu + \overline{T}^\mu S) \mathbf{A}_\mu \gamma^5] \bigg\}. \end{aligned} \quad (2.15)$$

The splittings  $\Delta M_S = M_S - M_H$  and  $\Delta M_T = M_T - M_H$  between the excited and ground state multiplets are independent of heavy quark flavor and do not vanish in the infinite quark mass limit. They are consequently included into the kinetic part of the zeroth order Lagrangian. In the interaction terms, the Goldstone fields couple to the mesons through the axial vector combination  $\mathbf{A}^\mu = i(\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger)/2$ . They also communicate via the vector field  $\mathbf{V}^\mu = (\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger)/2$  which resides within the covariant derivatives

$$\begin{aligned} D^\mu H &= \partial^\mu H - H(\mathbf{V}^\mu) \\ D^\mu S &= \partial^\mu S - S(\mathbf{V}^\mu) \\ D^\mu T^\nu &= \partial^\mu T^\nu - T^\nu(\mathbf{V}^\mu). \end{aligned} \quad (2.16)$$

The heavy meson chiral Lagrangian may be used to study strong interaction transitions among states within the  $H$ ,  $S$  and  $T^\mu$  superfields. We will focus upon the decays of P-wave  $D_s^{**}$  mesons in the following sections.

### 3. Kaon Decays of $D_s^{**}$ Mesons

The simplest  $D_s^{**}$  decay processes involve emission of a single Goldstone boson which must emerge in an even partial wave to conserve parity. Single pion decay of these  $I = 0$  states violates isospin, and eta decay is either severely phase space suppressed or kinematically forbidden. The single Goldstone boson which these strange charmed mesons therefore mainly eject is a kaon whose mass is comparable to the splittings between the excited and ground state multiplets.

If kinematically allowed, the strange members of the  $S$  multiplet predominantly decay through an  $\ell = 0$  partial wave down to states in  $H$  via the term proportional to  $f_1$  in (2.15). As guaranteed by heavy quark spin symmetry [15], the rates for the two charged kaon modes

$$\Gamma(D_{s0} \rightarrow D^0 K^+) = \frac{f_1^2}{4\pi} \left( \frac{M_{D^0}}{M_{D_{s0}}} \right) \frac{E_K^2 |\vec{p}_K|}{f_K^2} \quad (3.1a)$$

$$\Gamma(D'_{s1} \rightarrow D^{*0} K^+) = \frac{f_1^2}{4\pi} \left( \frac{M_{D^{*0}}}{M_{D'_{s1}}} \right) \frac{E_K^2 |\vec{p}_K|}{f_K^2} \quad (3.1b)$$

are equal to lowest order in the  $1/m_c$  expansion. The same is true for the neutral kaon decays  $D_{s0} \rightarrow D^+ K^0$  and  $D'_{s1} \rightarrow D^{*+} K^0$ . The equality among rates is broken however by formally subleading but phenomenologically important spin and flavor symmetry violating effects. We therefore choose to input actual meson masses into the kaon energies  $E_K = M_{D_s^{**}} - M_{D^{(*)}}$  and three-momenta  $|\vec{p}_K| = \sqrt{E_K^2 - M_K^2}$  in (3.1). We similarly set the Goldstone boson decay parameter  $f$  equal to  $f_K = 113$  MeV rather than  $f_\pi = 93$  MeV in the kaon decay rates.

Single Goldstone boson transitions between the  $T^\mu$  and  $H$  multiplets must proceed through an  $\ell = 2$  partial wave to conserve angular momentum. None of the dimension-4 terms in the leading order chiral Lagrangian can contribute to such processes. However at next-to-leading order, there exists a unique dimension-5 operator which does mediate D-wave decays:<sup>2</sup>

$$\mathcal{L}_v^{(1)} = \sum_{Q=c,b} \left\{ \frac{i\hbar}{\Lambda_\chi} \text{Tr} [ (\bar{H} T^\mu + \bar{T}^\mu H) \gamma^\nu \gamma^5 ] (D_\mu \mathbf{A}_\nu + D_\nu \mathbf{A}_\mu) + \dots \right\}. \quad (3.2)$$

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<sup>2</sup> There is no operator analogous to the one in (3.2) with the symmetric Goldstone expression  $D_\mu \mathbf{A}_\nu + D_\nu \mathbf{A}_\mu$  replaced by  $D_\mu \mathbf{A}_\nu - D_\nu \mathbf{A}_\mu$  since the antisymmetric combination identically vanishes. Consequently, there exists only one independent dimension-5 operator which mediates  $T^\mu \rightarrow H \boldsymbol{\pi}$  and not two as claimed in ref. [10].

Using the spin sums

$$\begin{aligned}
\sum_{I=1}^3 \epsilon_{\mu}^{(I)}(v)^* \epsilon_{\nu}^{(I)}(v) &= -g_{\mu\nu} + v_{\mu}v_{\nu} \\
\sum_{I=1}^5 \epsilon_{\mu\nu}^{(I)}(v)^* \epsilon_{\alpha\beta}^{(I)}(v) &= -\frac{1}{3}(g_{\mu\nu} - v_{\mu}v_{\nu})(g_{\alpha\beta} - v_{\alpha}v_{\beta}) \\
&\quad + \frac{1}{2}(g_{\mu\alpha} - v_{\mu}v_{\alpha})(g_{\nu\beta} - v_{\nu}v_{\beta}) + \frac{1}{2}(g_{\mu\beta} - v_{\mu}v_{\beta})(g_{\nu\alpha} - v_{\nu}v_{\alpha})
\end{aligned} \tag{3.3}$$

to average and sum over initial and final state polarizations, one finds the following rates for the allowed  $T^{\mu} \rightarrow HK$  transitions [10]:

$$\Gamma(D_{s1} \rightarrow D^*K) = \frac{5}{15\pi} \left( \frac{h}{f_K \Lambda_{\chi}} \right)^2 \left( \frac{M_{D^*}}{M_{D_{s1}}} \right) |\vec{p}_K|^5 \tag{3.4a}$$

$$\Gamma(D_{s2} \rightarrow DK) = \frac{2}{15\pi} \left( \frac{h}{f_K \Lambda_{\chi}} \right)^2 \left( \frac{M_D}{M_{D_{s2}}} \right) |\vec{p}_K|^5 \tag{3.4b}$$

$$\Gamma(D_{s2} \rightarrow D^*K) = \frac{3}{15\pi} \left( \frac{h}{f_K \Lambda_{\chi}} \right)^2 \left( \frac{M_{D^*}}{M_{D_{s2}}} \right) |\vec{p}_K|^5. \tag{3.4c}$$

As predicted by general spin symmetry arguments, these results occur in the ratio 5:2:3 in the infinite charm mass limit.

The coupling constant  $h$  multiplying the dimension-5 operator in (3.2) can be fixed from the decay rates of the  $J^P = 2^+$  flavor partners of  $D_{s2}$ . Their single pion and eta widths are simply related by  $SU(3)$  to the kaon expressions in (3.4b, c). We set the sum of the rates for  $D_2^0 \rightarrow D^+\pi^-$ ,  $D^0\pi^0$ ,  $D^0\eta$ ,  $D^{*+}\pi^-$  and  $D^{*0}\pi^0$  equal to the total width  $\Gamma(D_2^0) = 28_{-7}^{+8+6}$  MeV recently reported by CLEO [7]. Solving for  $h$  then yields the reasonable coupling constant value  $h/\Lambda_{\chi} = (0.23 \pm 0.04)/1000$  MeV. Once  $h$  is fixed, we can predict the width of  $D_{s2}$  as a function of its mass by summing the partial widths for  $D_{s2} \rightarrow D^0K^+$ ,  $D^+K^0$ ,  $D_s\eta$ ,  $D^{*0}K^+$  and  $D^{*+}K^0$ . The results for the central value and one-standard deviation of  $\Gamma(D_{s2})$  are plotted in fig. 1. As can be seen in the figure, the  $D_{s2}$  width lies in the 5-15 MeV range.

We should comment upon the uncertainties associated with our width prediction. The two basic ingredients that have gone into this result are  $SU(3)$  and heavy quark spin symmetry.  $SU(3)$  relates  $\Gamma(D_2^0 \rightarrow D\pi)$  to  $\Gamma(D_{s2} \rightarrow DK)$  and  $\Gamma(D_2^0 \rightarrow D^*\pi)$  to  $\Gamma(D_{s2} \rightarrow D^*K)$ . Spin symmetry on the other hand relates the partial widths  $\Gamma(D_2^0 \rightarrow DK)$  and  $\Gamma(D_2^0 \rightarrow D^*K)$ . The CLEO collaboration has recently reported an updated measurement for the ratio of the  $SU(3)$  analogues of these last two decay rates [7]:

$$R = \frac{\text{Br}(D_2^0 \rightarrow D^+\pi^-)}{\text{Br}(D_2^0 \rightarrow D^{*+}\pi^-)} = 2.1_{-0.6}^{+0.6+0.6}. \tag{3.5}$$



The HHCPT value  $R = 2.2$  for this ratio agrees remarkably well with the CLEO measurement and bolsters one confidence in the effective theory. Moreover if we simply adopt the experimental number in (3.5), then we do not in fact need to invoke spin symmetry to predict  $\Gamma(D_{s2})$ . Our width result consequently only depends upon  $SU(3)$ . Since this flavor symmetry is generally violated at the 30% level, we expect corrections of this order to  $\Gamma(D_{s2})$  beyond the one-standard deviation shown in fig. 1 which originates from the experimental uncertainty in  $\Gamma(D_2^0)$ . A more precise estimate for the magnitude of  $SU(3)$  corrections could in principle be determined by working at next-to-leading order in the chiral expansion. But unfortunately, a full subleading order analysis would introduce so many new operators with unknown coefficients that all predictive power would be lost.

We next turn to consider single Goldstone boson decays of axialvector P-wave mesons which are complicated by mixing among the  $J^P = 1^+$  states in the  $S$  and  $T^\mu$  multiplets. In the infinite heavy quark mass limit, the superfields' light angular momenta quantum numbers  $j_\ell = 1/2$  and  $j_\ell = 3/2$  are exact and preclude intermultiplet mixing. However for  $m_Q \neq \infty$ , the physical mass eigenstates are annihilated by linear combinations

$$\begin{aligned} Q_{i\mu}^L &= \cos \theta P'_{i\mu} + \sin \theta P_{i\mu} \\ Q_{i\mu}^H &= -\sin \theta P'_{i\mu} + \cos \theta P_{i\mu} \end{aligned} \tag{3.6}$$

of the axialvector operators inside  $S$  and  $T^\mu$ . We identify the observed  $D_1^0$ ,  $D_1^+$  and  $D_{s1}$  mesons with  $Q_1^H$ ,  $Q_2^H$  and  $Q_3^H$  respectively.

$D_{s1}$  is the only excited strange charmed meson which has been seen so far. Its width however has not yet been experimentally resolved, and only a 90% CL upper bound  $\Gamma \leq 2.3$  MeV on its total decay rate has been set [7]. We can use this limit to restrict the magnitude of the S-wave component to the meson's width. The constraint translates into an upper bound on the coefficient  $f_1$ . The results are listed in Table I for three different values of the mixing angle  $\theta$ :

$\theta$	$\Gamma_S(Q_3^H)_{\max} / \text{MeV}$	$\Gamma_D(Q_3^H) / \text{MeV}$	$(f_1)_{\max}$
$1^\circ$	2.166	0.134	3.82
$5^\circ$	2.167	0.133	0.77
$10^\circ$	2.170	0.130	0.38

Table I

As can be seen in the last column, the maximum limits on  $f_1$  are all of order unity. They are thus consistent with one's general expectations for a dimensionless coupling appearing in the leading order chiral Lagrangian.

With  $f_1$  constrained, we can use  $SU(3)$  symmetry to bound the ratio of S and D partial widths for the nonstrange  $D_1^0$  meson. In the past, mixing between the  $J^P = 1^+$  states in the  $S$  and  $T^\mu$  multiplets has been thought to induce a large S-wave component to the total width of the physical  $D_1^0$  mass eigenstate. Since the S-wave decay rate is significantly greater than the D-wave's, even a small  $\ell = 0$  admixture was believed to generate a sizable width enhancement and to account for most of the measured total width  $\Gamma(D_1^0) = 20_{-5}^{+6+3}$  MeV. (“A small grapefruit can be larger than a typical apple” [16].) Our bounds however for the S/D ratio lead to the opposite conclusion. (“A tiny grapefruit is smaller than a typical apple.”) As can be seen from the entries in Table II, the  $\ell = 2$  component of the total width actually dominates over the  $\ell = 0$  contribution for reasonable values of  $\theta$ :<sup>3</sup>

$\theta$	$\Gamma_S(Q_1^H)_{\max} / \text{MeV}$	$\Gamma_D(Q_1^H) / \text{MeV}$	$\Gamma_{\text{tot}}(Q_1^H)_{\max} / \text{MeV}$
$1^\circ$	3.35	7.41	10.76
$5^\circ$	3.36	7.35	10.71
$10^\circ$	3.36	7.19	10.54

Table II

The S-wave component therefore does not explain the  $2\sigma$  discrepancy between theory and experiment for the total  $D_1^0$  decay rate. The source of this disagreement remains poorly understood.

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<sup>3</sup> The CLEO and E687 collaborations have reported that they see no evidence for any S-wave contribution to  $D_1^0$  decay [7,9]. Their conclusion relies however upon the questionable assumption of no final state interactions.

#### 4. Pion Decays of $D_s^{**}$ Mesons

The rates for single kaon decay of  $D_s^{**}$  mesons depend critically upon the precise splittings among all the strange states in  $H$ ,  $S$  and  $T^\mu$ . At this time, only the mass  $M = 2535.1 \pm 0.6$  MeV of  $D_{s1}$  has been measured [7]. Assuming that its  $J^P = 2^+$  partner has an equal or greater mass, both of the states associated with  $T^\mu$  predominantly decay via kaon transitions. The situation for the strange members of the  $S$  multiplet however is less clear and more interesting since their masses are unknown. Several attempts have been made to estimate the energy levels of  $D_{s0}$  and  $D'_{s1}$  using the quark model.  $D_{s0}$  is generally predicted to be heavy enough so that kaon decay is kinematically possible. On the other hand, the variation in results from different quark model calculations for the  $D'_{s1}$  mass is sufficiently great that one cannot conclusively determine whether single kaon decay is allowed [17]. It is consequently possible that this strange meson must decay via other channels.

The first alternative to consider is the isospin violating transition  $D'_{s1} \rightarrow D_s^* \pi^0$ . The basic characteristics of this mode are very similar to those for  $D_s^* \rightarrow D_s \pi^0$  which has recently been studied in ref. [18]. It proceeds at tree level through emission of a virtual eta that subsequently mixes into a neutral pion. The intermediate  $\eta$  propagator effectively renders the amplitude inversely proportional to the strange quark mass which is smaller than a typical hadronic scale. The isospin violation factor associated with  $D'_{s1} \rightarrow D_s^* \pi^0$  [19]

$$(m_d - m_u)/(m_s - (m_u + m_d)/2) \simeq 1/43.7 \quad (4.1)$$

is therefore not so suppressed as one might have thought. Setting the Goldstone boson decay parameter  $f$  equal to  $f_\eta = 121$  MeV, we find the rate

$$\Gamma(D'_{s1} \rightarrow D_s^* \pi^0) = \frac{f_1^2}{32\pi} \left( \frac{M_{D_s^*}}{M_{D'_{s1}}} \right) \left[ \frac{m_d - m_u}{m_s - (m_u + m_d)/2} \right]^2 \frac{E_\pi^2 |\vec{p}_\pi|}{f_\eta^2} \quad (4.2)$$

for the single pion process.

Isospin conserving double pion emission represents the next most important decay mode for strange charmed P-wave mesons. Unlike their nonstrange counterparts [10],  $I = 0$   $D_s^{**}$  mesons cannot undergo double pion decay via pole graphs in which two  $I = 1$  pions are sequentially emitted. The two pions must instead emerge in an isospin zero combination from higher order terms in the chiral Lagrangian. At dimension-5, there exist

just two such operators which mediate the superfield transitions  $S \rightarrow H\pi\pi$  and  $T^\mu \rightarrow H\pi\pi$  and preserve heavy quark spin symmetry:

$$\mathcal{L}_v^{(1)} = \sum_{Q=c,b} \left\{ \frac{h_1}{\Lambda_\chi} \text{Tr}[(\bar{H}S + \bar{S}H)v^\mu \gamma^\nu] \text{Tr}[\mathbf{A}_\mu \mathbf{A}_\nu] + \frac{h_2}{\Lambda_\chi} \text{Tr}[(\bar{H}T^\mu + \bar{T}^\mu H)v^\nu] \text{Tr}[\mathbf{A}_\mu \mathbf{A}_\nu] + \dots \right\}. \quad (4.3)$$

At  $O(1/m_c)$ , additional spin symmetry violating operators can participate as well. We will focus however upon the leading terms in (4.3).

Working in the two pion center of mass frame, we can readily decompose the  $D_s^{**} \rightarrow D_s^{(*)}\pi\pi$  decay amplitudes into S-wave and D-wave components. After squaring the amplitudes, we find the following  $S \rightarrow H\pi\pi$  differential decay rates for these partial waves and their interference term:

$$d\Gamma(D_{s0} \rightarrow D_s^* \sum_{i=1}^3 \pi^i \pi^i)_{S,D,SD} = \frac{3}{2} \left( \frac{h_1}{f_\pi^2 \Lambda_\chi} \right)^2 (F_{S,D,SD}) d\Phi_3 \quad (4.4a)$$

$$d\Gamma(D'_{s1} \rightarrow D_s \sum_{i=1}^3 \pi^i \pi^i)_{S,D,SD} = \frac{3}{2} \left( \frac{h_1}{f_\pi^2 \Lambda_\chi} \right)^2 \left( \frac{1}{3} F_{S,D,SD} \right) d\Phi_3 \quad (4.4b)$$

$$d\Gamma(D'_{s1} \rightarrow D_s^* \sum_{i=1}^3 \pi^i \pi^i)_{S,D,SD} = \frac{3}{2} \left( \frac{h_1}{f_\pi^2 \Lambda_\chi} \right)^2 \left( \frac{2}{3} F_{S,D,SD} \right) d\Phi_3. \quad (4.4c)$$

For completeness, we also quote the corresponding  $T^\mu \rightarrow H\pi\pi$  differential widths even though they are in reality very small compared to the  $T^\mu \rightarrow HK$  rates in (3.4):

$$d\Gamma(D_{s1} \rightarrow D_s \sum_{i=1}^3 \pi^i \pi^i)_{S,D,SD} = \frac{3}{2} \left( \frac{h_2}{f_\pi^2 \Lambda_\chi} \right)^2 \left( \frac{2}{9} F_{S,D,SD} \right) d\Phi_3 \quad (4.5a)$$

$$d\Gamma(D_{s1} \rightarrow D_s^* \sum_{i=1}^3 \pi^i \pi^i)_{S,D,SD} = \frac{3}{2} \left( \frac{h_2}{f_\pi^2 \Lambda_\chi} \right)^2 \left( \frac{1}{9} F_{S,D,SD} \right) d\Phi_3 \quad (4.5b)$$

$$d\Gamma(D_{s2} \rightarrow D_s^* \sum_{i=1}^3 \pi^i \pi^i)_{S,D,SD} = \frac{3}{2} \left( \frac{h_2}{f_\pi^2 \Lambda_\chi} \right)^2 \left( \frac{3}{9} F_{S,D,SD} \right) d\Phi_3. \quad (4.5c)$$

As required by heavy quark spin symmetry, the differential widths in (4.4a, b, c) and (4.5a, b, c) occur in the ratios 3 : 1 : 2 and 2 : 1 : 3 respectively in the infinite charm mass limit [15].

The functions  $F_S$ ,  $F_D$  and  $F_{SD}$  entering into (4.4) and (4.5) may be conveniently expressed in a manifestly Bose symmetric form in terms of the energies  $E_1$  and  $E_2$  of the

two pions measured in the decaying  $D_s^{**}$  rest frame, the pion pair invariant mass  $s$ , and the mass splitting  $\Delta M$  between the initial and final heavy mesons:

$$\begin{aligned}
F_S &= \frac{1}{9} \frac{\Delta M^2}{s^2} (\Delta M^2 - s)(s + 2m_\pi^2)^2 \\
F_D &= \frac{1}{4} \left[ (\Delta M^2 - 4E_1 E_2)^2 + \left(1 - \frac{2}{3} \frac{\Delta M^2}{s}\right) (\Delta M^2 - 4E_1 E_2)(s - 4m_\pi^2) \right. \\
&\quad \left. + \frac{1}{9} \frac{\Delta M^2}{s^2} (\Delta M^2 - s)(s - 4m_\pi^2)^2 \right] \\
F_{SD} &= -\frac{1}{3} \frac{\Delta M^2}{s} (s + 2m_\pi^2) \left[ \Delta M^2 - 4E_1 E_2 - \frac{1}{3} \left( \frac{\Delta M^2}{s} - 1 \right) (s - 4m_\pi^2) \right].
\end{aligned} \tag{4.6}$$

The three body phase space factor can also be simply written in terms of these variables:

$$d\Phi_3 = \frac{1}{64\pi^3} \delta(\Delta M - E_1 - E_2) dE_1 dE_2 ds. \tag{4.7}$$

Recall that in a charm or bottom hadron decay, the heavy body in the final state must generally recoil in order to conserve momentum. However, it carries away no kinetic energy in the limit of its mass tending towards infinity. The energy conserving delta function in  $d\Phi_3$  therefore constrains the two final state pions to take away all of the kinetic energy released by the original  $D_s^{**}$  meson.

The double pion decay rates in (4.4) and (4.5) can be used to test basic HQET ideas. The extent to which these differential widths will agree or disagree with future experimental measurements provides some measure of the importance of  $O(1/m_c)$  spin-flavor violating effects. Since the presumption that the charm quark is truly heavy represents the weakest point in most HQET applications, it is important to test this hypothesis in as many different settings as possible. Our P-wave meson decay expressions provide such an opportunity.

The differential two pion rates may be integrated to obtain the corresponding total partial wave widths. Integrating the functions in (4.6) over  $s$  between its upper and lower limits for fixed  $E_1$  and  $E_2$

$$s_{\pm} = 2[m_\pi^2 + E_1 E_2 \pm \sqrt{(E_1^2 - m_\pi^2)(E_2^2 - m_\pi^2)}], \tag{4.8}$$

we find

$$\begin{aligned}
\int_{s_-}^{s_+} ds F_S &= \frac{4}{9} \Delta M^2 \left\{ [\Delta M^2 - 5m_\pi^2 - 2E_1 E_2] \sqrt{(E_1^2 - m_\pi^2)(E_2^2 - m_\pi^2)} \right. \\
&\quad \left. + m_\pi^2 (\Delta M^2 - m_\pi^2) \log \frac{m_\pi^2 + E_1 E_2 + \sqrt{(E_1^2 - m_\pi^2)(E_2^2 - m_\pi^2)}}{m_\pi^2 + E_1 E_2 - \sqrt{(E_1^2 - m_\pi^2)(E_2^2 - m_\pi^2)}} \right\} \\
\int_{s_-}^{s_+} ds F_D &= \frac{4}{9} \Delta M^2 \left\{ [\Delta M^2 - 2m_\pi^2 + (18 \frac{m_\pi^2}{\Delta M^2} - 8) E_1 E_2 + 18 \frac{(E_1 E_2)^2}{\Delta M^2}] \sqrt{(E_1^2 - m_\pi^2)(E_2^2 - m_\pi^2)} \right. \\
&\quad \left. + m_\pi^2 (\Delta M^2 - m_\pi^2 - 6E_1 E_2) \log \frac{m_\pi^2 + E_1 E_2 + \sqrt{(E_1^2 - m_\pi^2)(E_2^2 - m_\pi^2)}}{m_\pi^2 + E_1 E_2 - \sqrt{(E_1^2 - m_\pi^2)(E_2^2 - m_\pi^2)}} \right\} \\
\int_{s_-}^{s_+} ds F_{SD} &= -\frac{8}{9} \Delta M^2 \left\{ [\Delta M^2 + m_\pi^2 - 5E_1 E_2] \sqrt{(E_1^2 - m_\pi^2)(E_2^2 - m_\pi^2)} \right. \\
&\quad \left. + m_\pi^2 (\Delta M^2 - m_\pi^2 - 3E_1 E_2) \log \frac{m_\pi^2 + E_1 E_2 + \sqrt{(E_1^2 - m_\pi^2)(E_2^2 - m_\pi^2)}}{m_\pi^2 + E_1 E_2 - \sqrt{(E_1^2 - m_\pi^2)(E_2^2 - m_\pi^2)}} \right\}.
\end{aligned} \tag{4.9}$$

We perform the remaining integrations over pion energies numerically.

We should note that final state interactions have been neglected here. The OZI rule leads one to expect that such interactions between the strange heavy meson and the two pions are small. Furthermore, experimentally measured pion scattering phase shifts indicate that the interactions between the pions themselves are also small in the D-wave. However, S-wave pion final state interactions are known to be important over much of the kinematic regime for  $D_s^{**} \rightarrow D_s^{(*)} \pi \pi$  transitions. So although we expect our S-wave differential decay rates to be accurate for invariant dipion masses near  $s = 4m_\pi^2$ , its integrated rate cannot really be trusted to provide much more than an order of magnitude estimate. Final state interactions could be incorporated into our results by altering the S-wave amplitude calculated here to include the pion scattering phase shift in a manner consistent with unitarity [20].

We plot the total integrated double pion width for  $D'_{s1}$  as a function of its mass in fig. 2. The meson's single kaon and single pion decay rates are also illustrated for comparison. We have set the unknown coupling constant  $f_1$  which enters into  $\Gamma(D'_{s1} \rightarrow D^* K)$  and  $\Gamma(D'_{s1} \rightarrow D_s^* \pi^0)$  equal to unity. On the other hand, we have set  $h_1 = 0.1$  in  $\Gamma(D'_{s1} \rightarrow D_s^{(*)} \sum_i \pi^i \pi^i)$  since it is OZI suppressed. Given the substantial uncertainties in these couplings, the curves in fig. 2 provide only qualitative information. However, it is obvious from the figure that the kaon transition completely dominates over the pion processes if it is kinematically allowed. This is not surprising since the two body kaon decay

is mediated by a dimension-4 operator in the leading order chiral Lagrangian whereas the single pion mode violates isospin while the double pion transition involves three bodies in the final state and proceeds only at next-to-leading order. It is also likely that the  $D'_{s1}$  state will never be observed if single kaon emission is indeed kinematically possible since its width would be very broad. But if its mass turns out to be less than 2504 MeV, then the  $D'_{s1}$  width should be quite narrow. In this situation, it will hopefully be possible to observe this strange P-wave meson in the future through its single or double pion channels. Indeed, we believe this scenario represents the best prospect for ever finding any of the  $S$  multiplet mesons.

## 5. Conclusion

Our study of Goldstone boson decays of strange charmed P-wave mesons has yielded a number of results which can be experimentally tested. Our model independent prediction for the width of  $D_{s2}$  lies in an experimentally accessible range. We are consequently optimistic that this  $J^P = 2^+$  state will soon be observed. On the other hand, detecting the  $D'_{s1}$  partner of  $D_{s1}$  is much more uncertain. Single kaon emission must be kinematically forbidden in order for this  $D'_{s1}$  state to ever be seen. We believe however that it is worthwhile to search for this resonance through its single and double two pion decay modes.

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*Note added:* A few weeks after completion of this paper, the CLEO collaboration reported the discovery of the  $D_{s2}$ . Its measured mass and width are  $M_{D_{s2}} = 2573.2 \pm 1.9$  MeV and  $\Gamma_{D_{s2}} = 16^{+5+3}_{-4-3}$  MeV [21].

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## Figure Captions

- Fig. 1. Total predicted width of  $D_{s2}$  plotted as a function of its mass. The dashed curves lying above and below the solid curve delineate the one-standard deviation region about the width's central value. Additional uncertainties due to  $SU(3)$  violation are not pictured.
- Fig. 2. Single kaon, single pion and double pion  $D'_{s1}$  decay rates plotted as a function of the heavy meson's mass. The dashed curve denotes  $\Gamma(D'_{s1} \rightarrow D^{*0}K^+) + \Gamma(D'_{s1} \rightarrow D^{*+}K^0)$ , while the dotdashed curve corresponds to  $\Gamma(D'_{s1} \rightarrow D_s^*\pi^0)$ . The solid curve represents the sum of the widths  $\Gamma(D'_{s1} \rightarrow D_s \sum_i \pi^i \pi^i) + \Gamma(D'_{s1} \rightarrow D_s^* \sum_i \pi^i \pi^i)$ . We have assumed the dimensionless coupling constant values  $f_1 = 1.0$  and  $h_1 = 0.1$  in this plot.

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